a short section on the Hardy–Littlewood maximal function. Chapter 2 is where orthogonal systems in a Hilbert space enter the picture. Here we find the typical results on Gram matrices, Gram–Schmidt orthogonalization, and completeness. Orthogonal polynomials and their extremal properties are of course treated in more detail. In particular we find Christoffel's theorem regarding orthogonal polynomials with weight $\rho(x)\,d\mu(x)$ in terms of orthogonal polynomials with measure μ, where ρ is a nonnegative polynomial. The three-term recursion relation and related Jacobi matrices are given, with a proof of Favard's theorem based on the spectral theorem for self-adjoint operators. The reproducing kernel for polynomial approximation is referred to as the Dirichlet kernel in this book (the Christoffel–Darboux kernel would have been more appropriate). The Jacobi polynomials are then treated in some detail (explicit formulas, the recurrence relation, the differential equation, uniform estimates, asymptotic properties and weighted estimates). Next, the author explains different approaches to obtaining estimates for general orthogonal polynomials on an interval $[a, b]$ and their Christoffel functions, including an approach using the recursion relation.

Chapter 3 deals with convergence and summability of Fourier series. Convergence in norm, pointwise convergence, and almost everywhere convergence are considered (but apparently not the uniform convergence), together with estimates of the Lebesgue function. The chapter ends with the convergence of the arithmetic (Cesàro) means of the partial sums of the Fourier series. Chapter 4 looks at the Fourier series in L^p $(1<p<\infty)$ and in the space C of continuous functions. First it is shown that for each orthonormal system of polynomials, there is a continuous function such that its Fourier series is not uniformly convergent. Then some technical estimates of the Lebesgue function are proved and some results of the author on strong summability in L^p are presented. Fourier series in L^1 are the topic of Chapter 5, with Poisson–Abel summability, which culminates in the analog of Fatou's theorem on differentiated Fourier series. The final chapter contains work of the author on trilinear kernels, extending the Christoffel–Darboux kernel (or Dirichlet kernel) which is a bilinear kernel. This trilinear kernel is useful for finding (generalized) product formulas for orthogonal polynomials and for investigating a generalized translation operator in orthogonal polynomials.

Overall, this book treats Fourier series in orthogonal polynomials in a rather classical approach. Only orthogonal polynomials on a finite interval are considered. Laguerre and Hermite polynomials are not covered, and in general there is no treatment in this book of Fourier series in orthogonal polynomials on infinite intervals. The newer results of the author are usually quite technical. The English language is often very poor and one would expect the publisher to put more effort into helping out authors whose native language is not English.

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A workshop on *Applications and Computation of Orthogonal Polynomials* took place between March 22 and 28, 1998, at the Oberwolfach Mathematical Research Institute. This volume contains refereed versions of 18 papers presented at (or submitted to) the conference. The volume is dedicated to the memory of Günther Hämmerlin who initiated, directed, and co-directed a series of Oberwolfach conferences on numerical integration (a picture is included). Topics covered are Gauss quadrature, least-squares polynomial approximation, Szegő polynomials, Lanczos' method, Müntz orthogonal polynomials, and various papers dealing with classical and nonclassical orthogonal polynomials.

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